Trigonometry Mathematics Content Standards

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified before they are used with students.

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

2.0 Students know the definition of sine and cosine as *y*- and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

Find an angle β between 0 and 2π such that $\cos(\beta) = \cos(6\pi/7)$ and $\sin(\beta) = -\sin(6\pi/7)$. Find an angle θ between 0 and 2π such that $\sin(\theta) = \cos(6\pi/7)$ and $\cos(\theta) = \sin(6\pi/7)$.

3.0 Students know the identity $\cos^2(x) + \sin^2(x) = 1$:

3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).

3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

Prove $\csc^2 x = 1 + \cot^2 x$.

4.0 Students graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift.

On a graphing calculator, graph the function $f(x) = \sin(x) \cos(x)$. Select a window so that you can carefully examine the graph.

1. What is the apparent period of this function?
2. What is the apparent amplitude of this function?
3. Use this information to express f as a simpler trigonometric function.

5.0 Students know the definitions of the tangent and cotangent functions and can graph them.

6.0 Students know the definitions of the secant and cosecant functions and can graph them.

7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

Use the addition formula for sine to find a numerical value of $\sin(75°)$.

Use the addition formula to find the numerical value of $\sin(15°)$.

Is $g(x) = 5 \sin 3x + 2 \cos x$ a periodic function? If so, what is its period? What is its amplitude?

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

Express $\sin 3x$ in terms of $\sin x$ and $\cos x$.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

A vertical pole sits between two points that are 60 feet apart. Guy wires to the top of that pole are staked at the two points. The guy wires are 40 feet and 35 feet long. How tall is the pole?

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

Suppose in $\triangle ABC$ and $\triangle A'B'C'$, the sides of AB and $A'B'$ are congruent, as are AC and $A'C'$, but $\angle A$ is bigger than $\angle A'$. Which of $\triangle ABC$ and $\triangle A'B'C'$ has a bigger area? Prove that your answer is correct.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

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16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.

Express the circle of radius 2 centered at (2, 0) in polar coordinates.

17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

What is the angle that the ray from the origin to $3 + \sqrt{3}i$ makes with the positive x-axis?

18.0 Students know DeMoivre's theorem and can give nth roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.

A lighthouse stands 100 feet above the surface of the ocean. From what distance can it be seen? (Assume that the radius of the earth is 3,960 miles.)